COURSE SYLLABUS
COMPLEX ANALYSIS • SPRING 2020 • MATH 427/517

SIRAN LI

Lecture Information: MWF 1:00–1:50pm, Herman Brown Hall 453.

Course Description: The field of real numbers $\mathbb{R}$ is extended to the field of complex numbers $\mathbb{C}$ by introducing a root $i = \sqrt{-1}$ to the polynomial equation $x^2+1 = 0$. In this course, we study the analysis of functions of one complex variable. In contrast to what we have learned from real analysis, complex functions whose first derivative exists exhibit much more "rigid" behaviours; *e.g.*, it is automatically infinitely differentiable, with derivatives of all orders representable as suitable integrals over closed curves. We shall start with basic properties of complex numbers and quickly build up to more advanced topics including Riemann surfaces and Riemann mappings. The geometrical/topological perspectives shall be emphasised throughout.

Prerequisites: The students should have a solid knowledge about real analysis (both single and multi-variable) and point set topology (in particular, metric spaces). Knowledge about algebraic topology, differential geometry, and differential equations will be helpful, though not required.

Course Outline: Complex numbers; analyticity/holomorphicity; Cauchy–Riemann equations; line integrals; Cauchy's theorem over 0-homologous loops; Cauchy's integral formula and its consequences (Taylor expansion, Liouville's theorem, and the fundamental theorem of algebra); the argument principle and Rouché's theorem; classification of isolated singularities; modulus of entire functions; canonical infinite products and Hadamard's factorisation theorem.

Optional: Analytic continuation (sheaf-theoretic); algebraic curves and Riemann surfaces; conformal mappings; Riemann mapping theorem; modular functions; Picard's theorem.

Text: We shall mainly follow the classic by Ahlfors (3^{rd} ed.). For modern treatments, *cf.* Stein–Shakarchi. See also other great classical texts by Rudin, Kodaira, and Titchmarsh, etc.

Assessment: One set of about 30 problems of varying difficulty will be assigned during the first week. Roughly, solving about 1/2 of the assigned problems correctly and completely (no partial credits) will lead to a "B", and 2/3 will lead to an "A". In addition, you should hand in at least 1/3 of your work before the spring recess.

Office Hours: Monday and Wednesday 4–5pm in Herman Brown Hall 452, or by appointment via email <Siran.Li@rice.edu>. The TA for the course is Jerez Chen <Yikai.Chen@rice.edu>. Please also contact Jerez for (informal) advice on Ph.D. qualification exams.